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Short Papers

Suspended Coupled Slotline Using Double Layer Dielectric

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Abstract—**This paper presents a rigorous analysis of coupled slotline a) on a double-layer dielectric substrate, and b) sandwiched between two dielectric substrates. The dielectric substrates are of arbitrary thickness and permittivity and the structure is assumed to be suspended inside a conducting enclosure of arbitrary dimensions. The odd- and even-mode dispersion and characteristics impedance, along with the effect of shielding on these, are illustrated. These structures should find extensive applications in the fabrication of MIC components, such as directional couplers, phase shifters, and mixers.**

I. INTRODUCTION

Slotline on a dielectric substrate [1] is ideally suited for MIC components, such as directional couplers [2], phase shifters [3], and balanced mixers [4]. Alumina, which is the usual choice of the substrate material at X-band and below, is excessively dispersive at millimeter-wave frequencies. By reducing the substrate thickness dispersion is reduced, but this results in extremely narrow linewidth which increases line loss and makes reproduction difficult [5]. Recently, two new slotline structures have been proposed; namely, slotline on a double-layer dielectric substrate [6] and the sandwich slotline [7]. The disadvantages inherent in the conventional slotline are overcome in the double-layer slotline where an additional dielectric layer of low permittivity is introduced between the ground plane containing the slot and the bottom dielectric layer. The sandwich slotline, with top dielectric layer of low permittivity, is also useful at millimeter-wave frequencies. However, it has a slightly higher value of effective dielectric constant when compared with the double-layer slotline with same permittivities.

The paper analyzes firstly, the coupled slotline on a double-layer dielectric substrate suspended inside a conducting enclosure of arbitrary dimensions, and, secondly, the coupled slotline sandwiched between two dielectric substrates suspended inside a conducting enclosure of arbitrary dimension. For the sake of convenience the dielectric substrates are of arbitrary thickness and permittivity. Invariably a practical system is shielded from the environment in order to protect it from RF interference. Hence the study also illustrates the effect of shielding on the computed odd- and even-mode dispersion and characteristic impedance.

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II. ANALYSIS

A schematic diagram of the structures to be analyzed is shown in Fig. 1(a) and (b). For the case of odd excitation, a magnetic wall is placed at the $y=0$ plane (Fig. 1(c)); it then suffices to restrict the analysis to the right half of the structure. A similar simplification is possible for the case of even excitation except that the magnetic wall at the $y=0$ plane is replaced by an electric wall. Modifying Cohn's analysis of a slotline on a dielectric substrate [1], the odd- and even-mode structures can be easily reduced to an asymetric capacative iris backed by a double-layer dielectric substrate or sandwiched between two dielectric substrates in a rectangular waveguide (Fig. 1(d)). It then follows that for the odd excitation, the full set of modes satisfying the boundary conditions are the $TE_{1,n}$ and $TM_{1,n}(n \geqslant 0)$, and for the even excitation, $TE_{1,n}(n \geqslant 0)$ and $TM_{1,n}(n>0)$. The resulting expressions are given below for the odd mode only.

Let $p=\lambda/2a$ be an independent variable, where λ is the free-space wavelength and a is the length of the slot as in Cohn's analysis [1]. At the transverse resonance frequency $a=\lambda'/2$ and $p=\lambda/\lambda'$ for $B_t=0$, where λ' is the wavelength in the slotline and B_t is the total susceptance at the plane of the slot. Slotline on a double-layer dielectric (odd mode):

The total E_y and H_x fields at the $z=0$ plane and $x=a/2$ are functions of y as follows:

$$E_y = \sum_{n=0,1,2,\cdots}^{\infty} R_n \sin((2n+1)/2)\pi y/b \tag{1}$$

$$H_x = -\sum_{n=0,1,2,\cdots}^{\infty} y_{\text{in}} R_n \sin((2n+1)/2)\pi y/b. \tag{2}$$

Where the input wave admittance y_{in} is defined as in [1] and the

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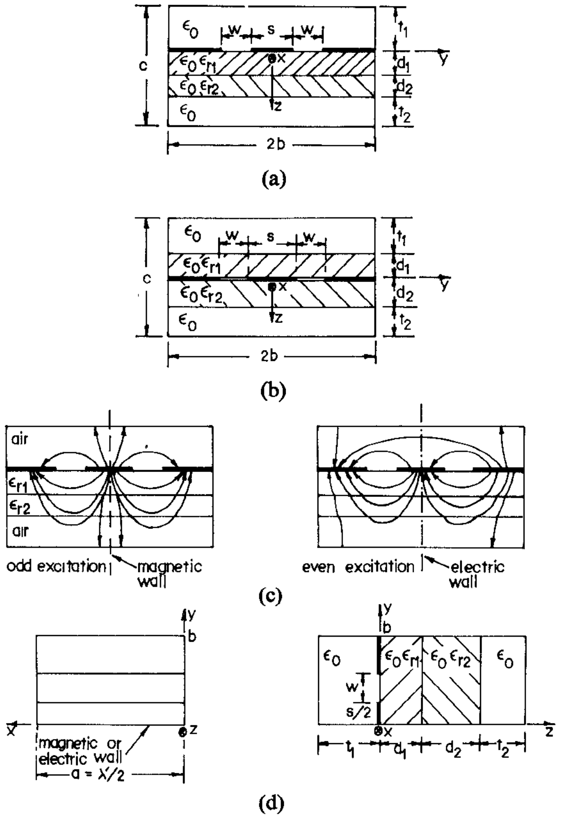

Fig. 1. (a) A schematic of shielded coupled slotline on a double-layer dielectric substrate. (b) A schematic of shielded coupled slotline sandwiched between two dielectric substrates. (c) The odd- and even-mode excitations of shielded coupled slotline on a double-layer dielectric substrate and insertion of magnetic or electric wall at $y=0$ plane. (d) Waveguide model for coupled slotline solution.

constant

$$R_n = 2C_0\,\delta\left[\frac{\sin\left(\frac{2n+1}{2}\right)\frac{\pi\delta}{2}}{\left(\frac{2n+1}{2}\right)\frac{\pi\delta}{2}}\cdot\sin\left(\frac{2n+1}{2}\right)\frac{\pi\bar{\delta}}{2}\right]. \tag{3}$$

The susceptance B_b looking to the right at the $z=0$ plane (Fig. 1(d)) is

$$jB_d = 2\sum_{n=0,1,2,\cdots}^{\infty} Y_{\text{in}}\left[\frac{\sin\left(\frac{2n+1}{2}\right)\frac{\pi\delta}{2}}{\left(\frac{2n+1}{2}\right)\frac{\pi\delta}{2}}\cdot\sin\left(\frac{2n+1}{2}\right)\frac{\pi\bar{\delta}}{2}\right]^2 \tag{4}$$

where Y_{in} is the admittance seen by all the higher order TE and TM modes directed into the dielectric-filled waveguide region of length d_1 and d_2 terminated in an air-filled region of length t_1. The susceptance B_a looking to the left at the $z=0$ plane (Fig. 1(d)) is obtained from (4) by letting $\epsilon_{r1}=\epsilon_{r2}=1$ or $d_1=d_2=0$. When B_d and B_a are added, the total susceptance B_t is obtained at the plane of the slot

$$\eta B_t = \frac{1}{p}\left\{(\epsilon_{r1}+1-2p^2)I + \sum_{n=0,1,2,\cdots}^{\infty}\left[v^2\left(1-\frac{\coth(m\pi F_n t_1/b)}{F_n}\right)+M_n\right]\cdot\frac{\sin^2(m\pi\delta/2)}{m(m\pi\delta/2)^2}\sin^2\frac{m\pi\bar{\delta}}{2}\right\} \tag{5}$$

where

$$m=(2n+1)/2$$
$$\eta=(\mu_0/\epsilon_0)^{1/2}=376.7\ \Omega$$
$$\delta=w/b$$
$$\bar{\delta}=(s+w)/b$$
$$v=(p^2-1)^{1/2}$$
$$u_1=(\epsilon_{r1}-p^2)^{1/2}$$
$$u_2=(\epsilon_{r2}-p^2)^{1/2}$$
$$F_n=\left[1+(bv/anp)^2\right]^{1/2}$$
$$F_{n1}=\left[1-(bu_1/anp)^2\right]^{1/2}$$
$$F_{n2}=\left[1-(bu_2/anp)^2\right]^{1/2}$$

$$I=\frac{1}{(\pi\delta)^2}\left\{-\frac{(\pi\bar{\delta})^2}{2}\ln\left(\frac{\pi\bar{\delta}}{4}\right)-\frac{(\pi\delta)^2}{2}\ln\left(\frac{\pi\delta}{4}\right)+\frac{\left[\pi(\bar{\delta}+\delta)\right]^2}{4}\ln\frac{\left[\pi(\bar{\delta}+\delta)\right]}{4}+\frac{\left[\pi(\bar{\delta}-\delta)\right]^2}{4}\ln\frac{\left[\pi(\bar{\delta}-\delta)\right]}{4}+\frac{3(\pi\delta)^2}{4}+\frac{3(\pi\bar{\delta})^2}{4}-\frac{3\left[\pi(\bar{\delta}+\delta)\right]^2}{8}-\frac{3\left[\pi(\bar{\delta}-\delta)\right]^2}{8}-\frac{(\pi\delta)^4}{576}-\frac{(\pi\bar{\delta})^4}{576}+\frac{\left[\pi(\bar{\delta}+\delta)\right]^4}{1152}+\frac{\left[\pi(\bar{\delta}-\delta)\right]^4}{1152}\right\}.$$

For F_{n1} and F_{n2} real, M_n is

$$M_n=\left\{\left[\epsilon_{r_1}\tanh(r_n)-p^2F_{n1}^2\coth(q_n)\right]/\left[1+(b+am)^2\right]F_{n1}\right\}-u_1^2 \tag{6}$$

where

$$r_n=(\pi m d_1F_{n1}/b)+\tanh^{-1}\left[(\epsilon_{r2}F_{n1}/\epsilon_{r1}F_{n2})\cdot\tanh\left\{(\pi m d_2F_{n2}/b)+\tanh^{-1}[(F_{n2}/\epsilon_{r2}F_n)\coth(\pi m t_2F_n/b)]\right\}\right]$$
$$q_n=(\pi m d_1F_{n1}/b)+\coth^{-1}\left[(F_{n2}/F_{n1})\coth\left\{(\pi m d_2F_{n2}/b)+\coth^{-1}[(F_n/F_{n2})\coth(\pi m t_2F_n/b)]\right\}\right].$$

Similarly for the sandwich slotline (odd mode):

$$\eta B_t=\frac{1}{p}\left\{(\epsilon_{r1}+\epsilon_{r2}-2p^2)I+\sum_{n=0,1,2,\cdots}^{\infty}(M_{n1}+M_{n2})\frac{\sin^2(m\pi\delta/2)}{m(m\pi\delta/2)^2}\sin^2(m\pi\bar{\delta}/2)\right\}. \tag{7}$$

For F_{ni} real, $i=1$ or 2

$$M_{ni}=\left\{\left[\epsilon_{ri}\tanh(r_{ni})-p^2F_{ni}^2\coth(q_{ni})\right]/\left[1+(b/am)^2\right]F_{ni}\right\}-u_i^2 \tag{8}$$

where

$$r_{ni}=(m\pi d_iF_{ni}/b)+\tanh^{-1}[(F_{ni}/\epsilon_{ri}F_n)\coth(m\pi F_nt_i/b)]$$
$$q_{ni}=(m\pi d_iF_{ni}/b)+\coth^{-1}[(F_n/F_{ni})\coth(m\pi F_nt_i/b)].$$

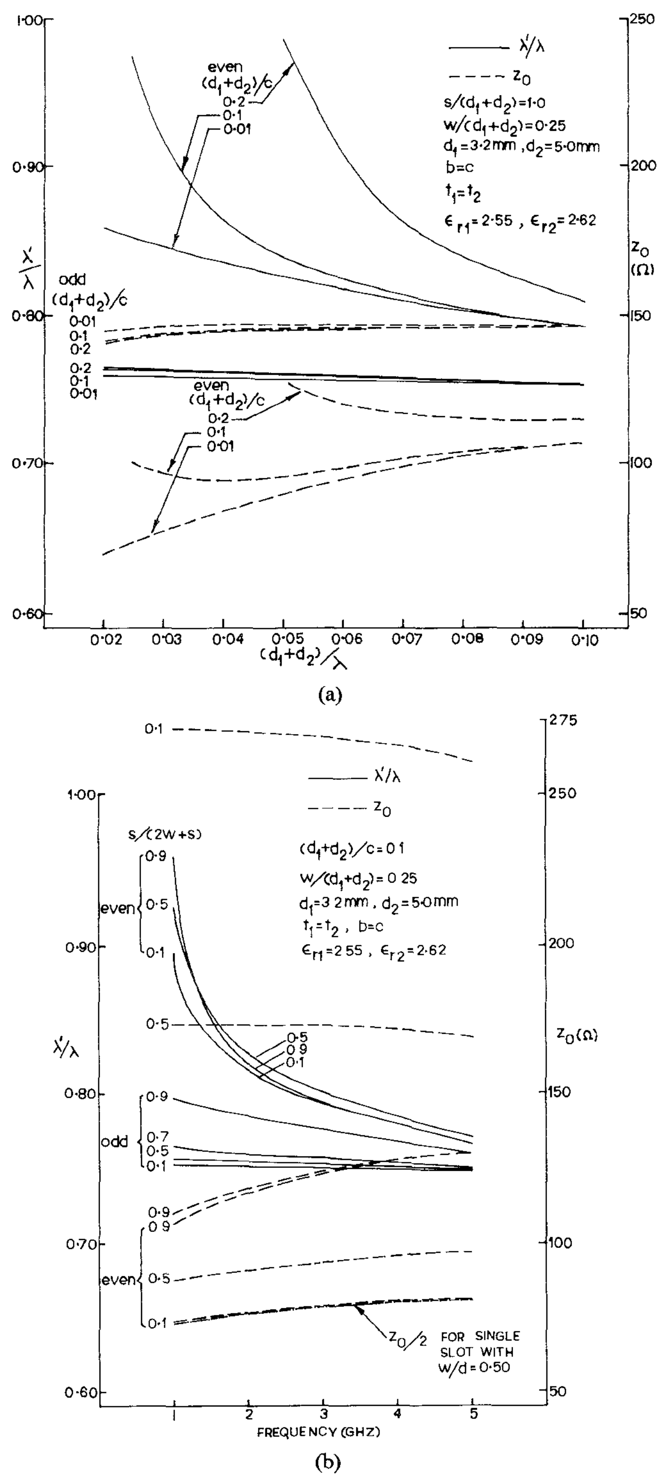

Fig. 2. (a) The effect of shielding on the odd- and even-mode dispersion and characteristic impedance of coupled slotline on a suspended double-layer dielectric substrate when the separation remains fixed. (b) The odd- and even-mode dispersion and characteristic impedance vis-à-vis the different frequencies and also the varying distance that separate the two slots when the height of the enclosure remains fixed.

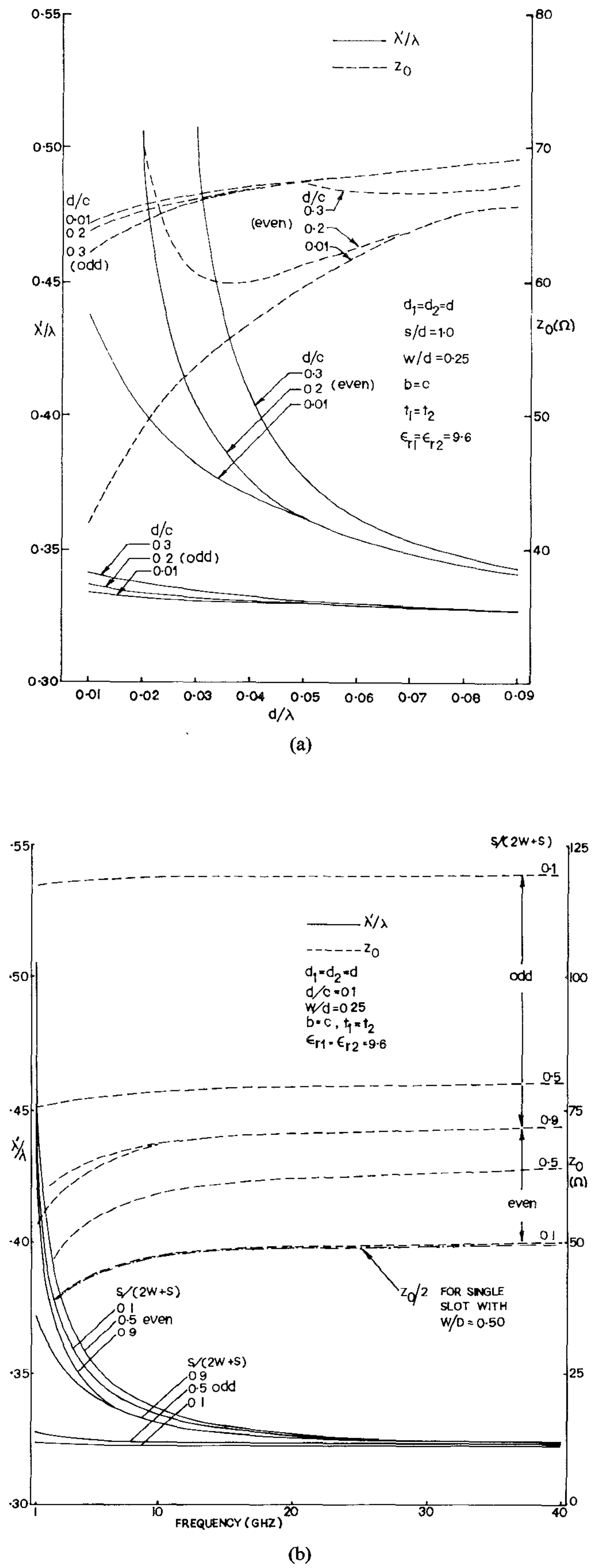

Fig. 3. (a) The effect of shielding on the odd- and even-mode dispersion and characteristic impedance of coupled sandwich slotline when the separation remains fixed. (b) The odd- and even-mode dispersion and characteristic impedance vis-à-vis the different frequencies and also the varying distance that separate the two slots when the height of the enclosure remains fixed.

The procedure in using the above equations to determine the dispersion and characteristic impedance is similar to that of Cohn [1] and is therefore not discussed here.

III. Numerical Results

The computed dispersion and characteristic impedance of both the structures are illustrated in Figs. 2 and 3, respectively. Figs. 2(a) and 3(a) illustrate the odd- and even-mode dispersion

TABLE I

Substrate Thickness	Open CPW, Static TEM Approximation, Davis [8]		Shielded CPW, Singular Integral Equation, Saha [9]	Odd-mode of Excitation	
	λ'/λ	$Zo(\Omega)$	λ'/λ	λ'/λ	$Zo(\Omega)$
Equal to the Slot-width(w)	0.48	57.7	0.443	0.496	55.7
Twice the slot width (2w)	0.45	-	0.439	0.452	51.0
Thrice the slot width (3w)	0.43	52.4	0.436	0.440	49.7

$\epsilon_{r1} = 10, \quad \epsilon_{r2} = 1.0$

$s/(2w+s) = 0.5, \; (d_1+d_2+t_2)/d_1 = 9, \; t_1 = 9w, \; b = 9w$

$k_o = 2\pi f\sqrt{\mu_o \epsilon_o}, \; k_o s = 0.02$

and also the characteristic impedance as a function of the normalized substrate thickness with the normalized height of the conducting enclosure as a parameter. It is observed that as the height of the conducting enclosure is gradually reduced, starting from a large value, the even mode slowly cuts off. On the other hand, the odd mode is almost insensitive to the height of the shielding enclosure.

Figs. 2(b) and 3(b) illustrate the computed odd- and even-mode dispersion and also the characteristics impedance, a function of the frequency. The slotlines are separated from each other by an amount $s/(2w+s)$. It is observed that for a fixed frequency and for small values of $s/(2w+s)$ the presence of the conductor between the slots has negligible effect on the even-mode propagation, and the two waves propagate as a single wave on a slotline with slot width $(2w+s)$. Hence the even mode λ'/λ is initially small. When $s/(2w+s)$ is increased, the slot width proportionally increases and the ratio λ'/λ also increases. When $s/(2w+s)$ continues to increase the two waves start to decouple and propagate as two independent waves on two slotlines. These two independent waves finally decouple totally. When each wave propagates on a slotline with slot width w, λ'/λ decreases. Furthermore, the computed even-mode characteristic impedance approaches one-half of the characteristic impedance of a single slotline. The width of the single slotline is twice that of the coupled structure. Nevertheless, the dispersion in both the cases is the same.

If $s/(2w+s)$ is small, and the slots are excited for the odd mode of propagation, then the effective slot width is smaller than w; and hence λ'/λ is the smallest. As the ratio $s/(2w+s)$ increases the effective slot width also increases, consequently λ'/λ increases. Finally, when $s/(2w+s)$ takes a large value the effective slot width moves nearer w; and, the ratio λ'/λ inclines toward the even mode λ'/λ. It may also be noted that the above structures are reduced to a coplanar waveguide (CPW) [8] for the odd mode of excitation. As a numerical check the odd-mode dispersion and characteristic impedance are computed and also compared with the results reported by Davis [8] and Saha [9] in Table I.

IV. Conclusion

Briefly, the paper presents an analysis of shielded coupled slotline a) on a double-layer dielectric substrate, and b) sandwiched between two dielectric substrates. It also illustrates the effect of shielding on the odd- and even-mode dispersion and characteristic impedance. The odd-mode dispersion and characteristic impedance agrees with the results reported by Davis [8] and Saha [9]. These structures should find extensive applications in the design of MIC components, such as, directional couplers, filters, phase shifters, and mixers.

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The Design of Broadside-Coupled Stripline Circuits

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***Abstract*—Accurate design expressions for broadside-coupled striplines are presented. The expressions have general validity as long as $W/S > 0.35$. Uncertainty analysis is described to calculate the effect of tolerances in parameters on coupling coefficient and input VSWR. The effect of tolerances in parameters on these characteristics increases as the coupling becomes tighter and tighter.**

I. Introduction

Broadside-coupled striplines have been used extensively in the design of many passive and active components, such as directional couplers, filters, baluns, and digital phase shifter networks. This configuration has been used widely for realizing tight couplings (e.g., 3-dB hybrids) because for greater than −8-dB coupling, the spacing between the strips in the case of parallel coupled transmission lines (i.e., striplines, microstrip lines, etc.) becomes prohibitively small.

Broadside-coupled striplines consist of two parallel strip conductors embedded in a dielectric between two ground planes as

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